

SFT on separated D-branes and D-brane translation

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Abstract

We discuss novel properties of the string field and the Open String Field Theory action arising in a system with multiple D-branes, then use the level truncation scheme to study marginal deformations and tachyon condensation in a system with two parallel but separated branes. We find string fields corresponding to D-brane decay combined with a finite change in the distance between the two D-branes. Using D-brane separation as a yardstick, we are able to continuously control the spacetime displacement of the D-branes and find that our solutions exist only for a finite range of this displacement. Thus, at least in level truncation, Open String Field Theory seems unable to describe the entire CFT moduli space.

Contents

1	Introduction	1
2	OSFT action for multiple D-branes	4
2.1	The real string field	4
2.2	Level truncation	7
2.3	The action	9
2.4	Two D-branes and exchange symmetry	11
3	D-brane decay	12
3.1	Level 0	12
3.2	Level (1,3)	13
3.3	Level (3,9)	15
3.4	Discussion	15
4	Restoration of SU(2) symmetry	21
	Appendix: The potential at higher levels	23

1 Introduction

String Field Theory has seen great progress in the last decade. Different classes of solutions in cubic SFT—tachyon condensation, marginal deformations and time dependent brane decay—have been understood through either numerical studies in level truncation [1–5] or by constructing exact analytic solutions [6–12], or both (for a review, see for example [13]). With a few exceptions (see for example [7, 14]) most of this work has focused on Open String Field Theory in the presence of a single D-brane. In this paper we begin an exploration of cubic OSFT in the presence of multiple D-branes. We find that the new degrees of freedom corresponding to off-diagonal components of the Chan-Paton matrices lead to new types of solutions. These solutions provide us with new tools to explore the structure and properties of cubic OSFT.

We work in bosonic string theory in 26 dimensions and focus on cubic OSFT in the presence of two parallel D24-branes extended in X_0, \dots, X_{24} directions and separated in the X_{25} direction, studying solutions to this SFT as a function of the separation of the two D24-branes. The presence of two D-branes, as opposed to just one, implies that each spacetime field of the string field is replaced by a 2×2 matrix of fields¹. This matrix is either hermitian or antihermitian (see Section 2.1 for more details). The diagonal elements of this matrix,

¹This is in some ways similar to the situation studied in [15], which focused on a separated brane-antibrane system. One key difference is the unbroken SU(2) symmetry present in our scenario at zero separation.

which we will call here the 11 and the 22 elements, correspond to strings that have both ends attached to either one of the two D-branes. In addition, the string field contains off-diagonal fields, 12 and 21, corresponding to strings stretched between the two D-branes. When the separation is zero, our SFT has a $SU(2)$ symmetry which is apparent in the action, while at non-zero separation this symmetry is broken and the distinction between diagonal and off-diagonal fields becomes important. For the 11 and the 22 fields, the action contains a copy of the single-brane OSFT action each. This, combined with the fact that the action has no terms linear in the off-diagonal fields 12 and 21, implies that all the already-known solutions in single-brane OSFT exist for the two-brane configuration at any D-brane separation (with all the 12 and the 21 fields simply set to zero). The cubic part of the action couples off-diagonal elements to the diagonal ones. Thus, when the off-diagonal elements 12 and 21 are nonzero there exist new solutions which have no analog in the single-brane theory. It is these solutions we set out to study, using a level truncation scheme. We find that these solutions present an interesting interplay between tachyon condensation and marginal deformations.

Since we have two D-branes, we can allow one to decay while the other does not. For zero separation, there is a full $SU(2)$ family of such solutions [7]. For non-zero separation, with a broken $SU(2)$ symmetry, we would expect only those solutions where either the 11 or the 22 tachyon field develops a vev to exist, corresponding to the decay of either the first or the second D-brane. However, the 11 and the 22 sectors of the string field each contain a massless string state whose vev corresponds to a marginal deformation of the worldsheet Conformal Field Theory that can be interpreted as displacing the D-branes from their original position in the X_{25} direction (T-dual to the Wilson line in that direction). At any D-brane separation, there should exist a string field with non-zero massless string field vevs that physically corresponds to the two D-branes coming together to zero separation. Once in this new configuration, should one of the D-branes decay, the decay would be happening at the $SU(2)$ -symmetric point. Thus we would expect a full $SU(2)$ family of solutions that represent a combination of such D-brane translation with tachyon condensation. However, as it is not possible to fully restore the $SU(2)$ symmetry in a level truncated model, we obtain instead isolated solutions in which the off-diagonal 12 and 21 tachyon elements develop a non-zero vev together with the 11 and the 22 massless string elements.

Thus, we find a family of solutions (one solution at every D-brane separation) in which the off-diagonal tachyon field is non-zero. At zero D-brane separation, the vevs of the massless modes are zero, and our solution corresponds to a $SU(2)$ rotation of a diagonal solution in which one of the two D-branes has decayed. At non-zero separation, the massless string state vevs are non-zero and increase with increased D-brane separation. Thus, we can interpret our solutions as a combination of a marginal deformation (D-brane translation) and D-brane decay. These solutions have an energy comparable to that of a single D-brane decay.

Our construction should be compared to that in [3, 4]. There, once a finite vev for the field corresponding to the massless string state was fixed, no solutions were found in

level truncation; one equation of motion was left unsatisfied. We are able to find such solutions because in the untruncated theory there exists a continuously parametrized family of solutions, with an isolated solution point surviving the truncation. By changing the original separation of the D-branes, we are able to find solutions to the equations of motion in which the deformation parameter has a finite vev which can be continuously adjusted. The price we pay for this is the presence of tachyon condensation which makes the marginal part of the deformation harder to isolate and analyze (see Section 3.4 for details).

The presence of two D-branes in the picture gives us an important tool: the physical separation of the D-branes is known and adjustable, thus it can be used as a yardstick for measuring the physical effect of the SFT deformation. Previous work on computing the CFT vev corresponding to a given SFT vev includes [4], which compared the energy-momentum tensors in the CFT and the SFT approaches, and [16], where lowest order corrections to the field redefinition between SFT fields and worldsheet fields were found. The solutions we find allow us to directly measure the vev of the conformal field theory parameter (i.e., the physical position of the D-brane) as a function of the deformation parameter in SFT. The presence of tachyon condensation means that the deformation parameter in our setup is not directly related to the vev of the SFT marginal parameter used in [3], but the problem of connecting them has been reduced to one purely within the SFT of one D-brane and is left for future work. A different approach to this problem is described in Section 4.

We show that there exists a maximum separation between the D-branes, of order $\sqrt{\alpha'}$, beyond which no deformation in OSFT is able to bring the two D-branes back together, suggesting that it is not possible to cover the entire moduli space of CFTs with OSFT in a single coordinate system. We see a mechanism similar to that in [3, 4], where two branches of OSFT solutions merge with no real solutions in existence beyond the point at which they meet. Note that because our adjustable parameter is the distance between the D-branes and not the infinitesimal marginal parameter for brane translation, our conclusions are not affected by the possibility that the marginal parameter might not parametrize the marginal trajectory in string field space beyond some finite distance.

This paper is organized as follows: In Section 2.1, we describe the properties of the string field in a scenario with N parallel D24-branes. We discuss in detail the reality condition, twist and Siegel gauge. We find that both the reality condition and the twist even condition lead to novel consequences once multiple D-branes are present. For example, the twist even condition is not equivalent to even level. In Section 2.2 we discuss level truncation, in Section 2.3 we construct the OSFT action and in Section 2.4 we discuss properties of the action peculiar to the set-up with just 2 D-branes. In Section 3, we present our solutions corresponding to a combination of a marginal deformation and D-brane decay and discuss their interpretation. Finally, in Section 4, we briefly discuss an attempt to restore $SU(2)$ symmetry using a purely marginal deformation to bring separated D-branes back to coincidence. We use $\alpha' = 1$ convention throughout the paper, except where we restore α' explicitly for clarity.

2 OSFT action for multiple D-branes

In this section, we study the string field and the OSFT action in a scenario with multiple D24-branes allowing for nonzero separation and for zero-mode fluctuations in the transverse direction. We find that the construction of a real, twist even, gauge fixed string field is more involved than in the case of a single D-brane. The complications are caused by several new elements: the string field has more than one vacuum (since there is one in every combination of Chan-Patton factors), twist acts nontrivially on these multiple vacua, and the zero mode in the transverse spacial direction takes different values depending the vacuum state. Our analysis would be equally applicable to other SFT scenarios where some (or all) of these elements are present.

2.1 The real string field

Consider a stack of N D24-branes, with X^{25} as the transverse direction. The mode expansion for the 25^{th} coordinate of a string starting on brane i and ending on brane j is

$$X^{25} = y_i - \frac{1}{2\pi i}(y_j - y_i) \ln(z/\bar{z}) + i\sqrt{\frac{1}{2}} \sum_{m \neq 0} \frac{\alpha_m^{25}}{m} \left(\frac{1}{z^m} - \frac{1}{\bar{z}^m} \right), \quad (1)$$

where the 25^{th} dimension is non-compact and y_i are the positions of D24-branes in the 25^{th} dimension. α_0^{25} can be nonzero when acting on a string field living on separated D-branes, when acting in a particular ij sector, we have:

$$\alpha_0^{25} \rightarrow d_{ij} \stackrel{\text{def}}{=} -\frac{y_j - y_i}{\pi\sqrt{2}}. \quad (2)$$

We assume standard mode expansions in the other 25 directions and in the ghost sector. Since the 25^{th} direction is non-compact, there are no winding modes. We will take $p_\mu = 0$ for $\mu = 0, \dots, 24$ because we are interested only in translationally invariant configurations. Therefore, the string field is built by acting with the oscillators α_n^μ , c_n and b_n on the zero-momentum ground states of strings stretching from brane i to brane j . We will denote the ground states with $|ij\rangle$ and normalize them so that $\langle ij|kl\rangle = \delta_{il}\delta_{jk}$ (i.e. $(|ij\rangle)^\dagger = \langle ji|$).

Consider then a matrix-valued spacetime field A of the string field, $\sum_{ij} A_{ij} \mathcal{A}|ij\rangle$, where \mathcal{A} is an operator built out of α_n^μ , c_n and b_n . For the string field to be real, it must be invariant under the combination of bpz and hermitian conjugation [17]. Therefore, if β_A , defined by $(bpz(\mathcal{A}))^\dagger = \beta_A \mathcal{A}$, is $+1$, the matrix of fields A is hermitian, $A_{ij} = \overline{A_{ji}}$. If $\beta_A = -1$, A must be anti-hermitian, $A_{ij} = -\overline{A_{ji}}$. We will refer to the first class of spacetime fields as ‘real’ and the second class as ‘imaginary’.

To check that the quadratic part of the action, proportional to $\langle \Phi | Q_B \Phi \rangle$, is real under this hermiticity condition, let the string field $|\Phi\rangle$ contain a term $\sum_{ij} A_{ij} \mathcal{A}|ij\rangle$ and $|Q_B \Phi\rangle$ contain

$\sum_{ji} B_{ji} \mathcal{B}|ji\rangle$, with matrix-valued fields A and B . Now $\langle \Phi | Q_B \Phi \rangle$ will contain two cross terms

$$\sum_{ij} A_{ij} B_{ji} (\beta_A \langle ij | \mathcal{A}^\dagger \mathcal{B} | ji \rangle) + \sum_{ij} A_{ij} B_{ji} (\beta_B \langle ji | \mathcal{B}^\dagger \mathcal{A} | ij \rangle) \quad (3)$$

which, with $\beta_A A_{ij} = (A^\dagger)_{ij} = \overline{A_{ji}}$ and $\beta_B B_{ji} = (B^\dagger)_{ji} = \overline{B_{ij}}$, can be combined to give

$$\begin{aligned} \sum_{ij} \beta_A (A_{ij} B_{ji} \langle ij | \mathcal{A}^\dagger \mathcal{B} | ji \rangle) + \sum_{ij} \beta_A (\overline{A_{ji} B_{ij}} \langle ji | \mathcal{B}^\dagger \mathcal{A} | ij \rangle) = \\ \sum_{ij} 2\beta_A \text{Re} (A_{ij} B_{ji} \langle ij | \mathcal{A}^\dagger \mathcal{B} | ji \rangle) , \end{aligned} \quad (4)$$

which is real as required.

Let us now check that the cubic part of the action is real as well. The cubic part of the action is proportional to $\langle \Phi | \Phi * \Phi \rangle$ and can be written in terms of the three-string vertex $\langle V_3 |$ as

$$\langle V_3 | \Phi^{(1)} \rangle | \Phi^{(2)} \rangle | \Phi^{(3)} \rangle = \langle \Phi | \Phi * \Phi \rangle . \quad (5)$$

Consider the twist symmetry, Ω , which reverses the orientation of the string. The three-string vertex is invariant under Ω . Notice that the mode expansion in (1) implies $\Omega \alpha_n^{25} \Omega^{-1} = (-1)^{n+1} \alpha_n^{25}$ and, in particular, $\Omega(\alpha_0^{25}) = -\alpha_0^{25}$. This is consistent with the fact that $\Omega|ij\rangle = -|ji\rangle$ (recall that the one-D-brane is twist odd, so that $\Omega|ii\rangle = -|ii\rangle$). Now, consider three terms of a string field: $\mathcal{A}|ij\rangle$, $\mathcal{B}|jk\rangle$, and $\mathcal{C}|ki\rangle$, such that $\Omega \mathcal{W} \Omega^{-1} = \Omega_W \mathcal{W}$ for $\mathcal{W} = \mathcal{A}, \mathcal{B}, \mathcal{C}$. Define

$$g(d_{ij}, d_{jk}, d_{ki}) \stackrel{\text{def}}{=} \langle V_3 | \mathcal{A}|ij\rangle \mathcal{B}|jk\rangle \mathcal{C}|ki\rangle . \quad (6)$$

Using the general properties of the string vertex, together with the fact that the string field is Grassmann odd and that the vacuum is twist odd, it is easy to show that

$$\langle V_3 | \mathcal{C}|ik\rangle \mathcal{B}|kj\rangle \mathcal{A}|ji\rangle = -\Omega_A \Omega_B \Omega_C g(d_{ij}, d_{jk}, d_{ki}) . \quad (7)$$

Trivially, it is also true that

$$\langle V_3 | \mathcal{B}|jk\rangle \mathcal{C}|ki\rangle \mathcal{A}|ij\rangle = \langle V_3 | \mathcal{C}|ki\rangle \mathcal{A}|ij\rangle \mathcal{B}|jk\rangle = g(d_{ij}, d_{jk}, d_{ki}) . \quad (8)$$

If the total string field contains these three terms with spacetime fields A_{ij} , B_{jk} and C_{ki} :

$$|\Phi\rangle = \dots + \sum_{ij} A_{ij} \mathcal{A}|ij\rangle + \sum_{jk} B_{jk} \mathcal{B}|jk\rangle + \sum_{ki} C_{ki} \mathcal{C}|ki\rangle + \dots , \quad (9)$$

then the cubic part of the action, $\langle \Phi | \Phi * \Phi \rangle$, contains a term

$$3 \sum_{ijk} g(d_{ij}, d_{jk}, d_{ki}) (A_{ij} B_{jk} C_{ki} - \Omega_A \Omega_B \Omega_C C_{ik} B_{kj} A_{ji}) . \quad (10)$$

Now, write all three fields \mathcal{A} , \mathcal{B} and \mathcal{C} in terms of the creation operators α_{-n} , c_{-m} , b_{-k} ($n > 0$, $m > -2$, $k > 1$). Looking at the following summary of the behaviour of different oscillators under the combined bpz and hermitian conjugation and under twist:

$$\begin{aligned} \Omega \alpha_n \Omega^{-1} &= (-1)^{n+1} \alpha_n & (bpz(\alpha_n))^\dagger &= (-1)^{n+1} \alpha_n \\ \Omega c_n \Omega^{-1} &= (-1)^n c_n & (bpz(c_n))^\dagger &= (-1)^{n+1} c_n \\ \Omega b_n \Omega^{-1} &= (-1)^n b_n & (bpz(b_n))^\dagger &= (-1)^n b_n \end{aligned} \quad (11)$$

we see that $\Omega_A = \beta_A (-1)^{N_c} (-1)^{N_F(N_F-1)/2}$ where N_c is the number of c_n -oscillators in the field \mathcal{A} and N_F is the number of fermionic oscillators (the last factor comes from the properties of the bpz conjugate while acting on a product of fermionic oscillators [18]). Since the field has ghost number 1, the number of b_n oscillators must be $N_c - 1$ and therefore $N_F = 2N_c - 1$. This implies that $\Omega_A = -\beta_A$ (which means that spacetime fields are hermitian (anti-hermitian) matrices if the twist eigenvalue of associated state \mathcal{A} is negative (positive)). We can rewrite equation (10) as

$$3 \sum_{ijk} g(d_{ij}, d_{jk}, d_{ki}) (A_{ij} B_{jk} C_{ki} + \beta_A \beta_B \beta_C C_{ik} B_{kj} A_{ji}) . \quad (12)$$

Using $\beta_A A_{ji} = (A^\dagger)_{ji} = \overline{A_{ij}}$ (and similarly for B and C) we see that the above expression is always real, as required, and equal to

$$6 \sum_{ijk} g(d_{ij}, d_{jk}, d_{ki}) \text{Re} (A_{ij} B_{jk} C_{ki}) . \quad (13)$$

As is usual in tachyon condensation computations, we restrict ourselves to twist even string fields. However, since the twist Ω acts in a non-diagonal way on the $|ij\rangle$ basis ($\Omega|ij\rangle = -|ji\rangle$), restricting to twist even is not the same as restricting to even level fields. To restrict our solutions to twist even fields, we can act on the string field with $\frac{1}{2}(1 + \Omega)$. In case of a real spacetime field corresponding to an operator \mathcal{A} , with $-\Omega_A = \beta_A = +1$ we obtain

$$\frac{1}{2}(1 + \Omega) A_{ij} \mathcal{A}|ij\rangle = \frac{1}{2}(A_{ij} + A_{ji}) \mathcal{A}|ij\rangle \quad (14)$$

and therefore we can restrict ourselves to a matrix of fields A which is not only hermitian, but also symmetric and therefore has real entries. In case of an imaginary spacetime field, $-\Omega_A = \beta_A = -1$ we we have

$$\frac{1}{2}(1 + \Omega) A_{ij} \mathcal{A}|ij\rangle = \frac{1}{2}(A_{ij} - A_{ji}) \mathcal{A}|ij\rangle , \quad (15)$$

which means we can restrict ourselves to an antihermitian antisymmetric matrix A , which again has purely real entries.

In the twist even sector, we wish now to impose Siegel gauge. Since this gauge can be imposed level by level for the usual reasons, we can do so at all levels except for level one,

where $L_0 = 0$ and the usual argument for local validity of Siegel gauge fails. We therefore will impose Siegel gauge starting at level two, and will include the level one state $c_0|ij\rangle$.²

We are now ready to write down the real, twist even, (mostly) Siegel gauge string field. We can use rotational invariance in the directions parallel to the D24-branes to argue that α_{-n}^μ oscillators for $\mu = 0, \dots, 24$ can appear only in combinations of the form $\alpha_{-n} \cdot \alpha_{-m} = \sum_{\mu=0}^{24} \alpha_{-n}^\mu \alpha_{-m}^\mu$. We single out the oscillators in the direction normal to the D24-branes by a superscript 25. L_{-n}^{25} are matter Virasoro generators for the CFT associated with the field X^{25} , while L'_{-n} are matter Virasoro generators for the CFT associated with the fields X^μ (with central charge 25), so that $L_{-n}^m = L_{-n}^{25} + L'_{-n}$. With this notation, the total string field up to level three is given in Tables 1 and 2. Table 1 uses the Virasoro generators in the matter sector, while Table 2 uses the α_{-n} oscillators. The relationship between the fields corresponding to these two different sets of states is given at the bottom of Table 2. We include the presentation in the α_{-n} basis because, at some D-brane separations, the Virasoro basis ceases to be complete. For example, at $d_{ij} = \pm 1/\sqrt{2}$ the states $L_{-2}^{25}c_1|ij\rangle$ and $L_{-1}^{25}\alpha_{-1}^{25}c_1|ij\rangle$ are no longer linearly independent. This degeneracy implies that the relationship between the fields in these two bases is singular at these special separations: at $d_{ij} = \pm 1/\sqrt{2}$, the relationship between (f_{ij}, w_{ij}) and $(\tilde{f}_{ij}, \tilde{w}_{ij})$ takes on a singular form $\tilde{w}_{ij} = w_{ij}/2 + f_{ij}/\sqrt{2}$, $\tilde{f}_{ij} = w_{ij}/\sqrt{2} + f_{ij}$, and (f_{ij}, w_{ij}) becomes infinite while $(\tilde{f}_{ij}, \tilde{w}_{ij})$ remains finite. Since we will never use a D-brane separation which is exactly equal to one of these special values, we will perform the computations in the Virasoro-generated basis. However, we will use some of the fields from the other basis to present our results, as these remain finite everywhere.

2.2 Level truncation

Following [5], we should define the level of the string field to include the total L_0 eigenvalue, and not just that part of it which counts oscillator excitations. Specifically, for a string stretched between D-branes i and j , we have

$$L_0 = \frac{1}{2}(d_{ij})^2 + \tilde{N} , \quad (16)$$

where \tilde{N} is the contribution to L_0 from the non-zero matter and the ghost oscillators. When acting on the diagonal part of the tachyon field, L_0 gives $L_0 c_1|ii\rangle = -c_1|ii\rangle$, and therefore we should perhaps define the level to be

$$l = \frac{1}{2}(d_{ij})^2 + (\tilde{N} + 1) \quad (17)$$

²For separated D-branes, we could have imposed Siegel gauge in the off-diagonal sector at level one, setting $c_0|ij\rangle$ to zero as long as $i \neq j$, but we will not find this to be necessary, since for two D-branes, the exchange symmetry described in section 2.4 will allow us to set the corresponding fields to zero anyway.

Level	String field $ \Phi\rangle$
0	$t_{ij}c_1 ij\rangle$
1	$(h_{ij}c_0 + x_{ij}\alpha_{-1}^{25}c_1) ij\rangle$
2	$(u_{ij}c_{-1} + v_{ij}L'_{-2}c_1 + w_{ij}L_{-2}^{25}c_1 + f_{ij}L_{-1}^{25}\alpha_{-1}^{25}c_1) ij\rangle$
3	$((o_1)_{ij}b_{-2}c_{-1}c_1 + (o_2)_{ij}c_{-2} + r_{ij}\alpha_{-1}^{25}c_{-1} + s_{ij}L'_{-2}\alpha_{-1}^{25}c_1 + p_{ij}L'_{-3}c_1 + q_{ij}L_{-3}^{25}c_1 + y_{ij}L_{-2}^{25}\alpha_{-1}^{25}c_1 + z_{ij}L_{-1}^{25}L_{-1}^{25}\alpha_{-1}^{25}c_1) ij\rangle$

Table 1: Level three string field in (mostly) Siegel gauge. $t, x, u, v, w, r, s, y, z$ are real symmetric matrices while h, f, o_1, o_2, p, q are real antisymmetric ones.

Level	String field $ \Phi\rangle$
0	$t_{ij}c_1 ij\rangle$
1	$(h_{ij}c_0 + x_{ij}\alpha_{-1}^{25}c_1) ij\rangle$
2	$(u_{ij}c_{-1} + \tilde{v}_{ij}\alpha_{-1} \cdot \alpha_{-1}c_1 + \tilde{w}_{ij}\alpha_{-1}^{25}\alpha_{-1}^{25}c_1 + \tilde{f}_{ij}\alpha_{-2}^{25}c_1) ij\rangle$
3	$((o_1)_{ij}b_{-2}c_{-1}c_1 + (o_2)_{ij}c_{-2} + r_{ij}\alpha_{-1}^{25}c_{-1} + \tilde{s}_{ij}\alpha_{-1} \cdot \alpha_{-1}\alpha_{-1}^{25}c_1 + p_{ij}\alpha_{-1} \cdot \alpha_{-2}c_1 + \tilde{q}_{ij}\alpha_{-1}^{25}\alpha_{-2}^{25}c_1 + \tilde{y}_{ij}\alpha_{-1}^{25}\alpha_{-1}^{25}\alpha_{-1}^{25}c_1 + \tilde{z}_{ij}\alpha_{-3}^{25}c_1) ij\rangle$

$$\tilde{v}_{ij} = \frac{1}{2}v_{ij} \quad \tilde{w}_{ij} = \frac{1}{2}w_{ij} + f_{ij}d_{ij} \quad \tilde{f}_{ij} = f_{ij} + w_{ij}d_{ij} \quad \tilde{s}_{ij} = \frac{1}{2}s_{ij}$$

$$\tilde{q}_{ij} = q_{ij} + y_{ij}d_{ij} + 3z_{ij}d_{ij} \quad \tilde{y}_{ij} = \frac{1}{2}y_{ij} + z_{ij}(d_{ij})^2 \quad \tilde{z}_{ij} = 2z_{ij} + y_{ij} + q_{ij}d_{ij}$$

Table 2: Level three string field in (mostly) Siegel gauge, written using α_{-n} oscillators instead of L_n . $t, x, u, \tilde{v}, \tilde{w}, r, \tilde{s}, \tilde{y}, \tilde{z}$ are real symmetric matrices while $h, \tilde{f}, o_1, o_2, p, \tilde{q}$ are real antisymmetric ones.

Then, once d_{ij} is large enough, we would need to include fields with higher \tilde{N} before including off-diagonal fields with lower \tilde{N} . For example, with two D-branes, diagonal fields with $\tilde{N} = 4$ should be included before off-diagonal fields with $\tilde{N} = 3$ if the D-brane separation d_{12} is greater than $\sqrt{2}$. However, since we will be studying the solutions as functions of the D-brane separation, changing which fields are included would cause discontinuities, making our results hard to interpret. We will therefore keep the field content of our truncated string field the same at all separations and, for simplicity, we will refer to $(\tilde{N} + 1)$ as the level. However, we should keep in mind that, for two D-branes, potentially more accurate results could be obtained for $d_{ij} > \sqrt{2}$ by adapting the field content accordingly.

2.3 The action

The OSFT potential (in units of 24-dimensional D-brane tension) can be expressed as

$$f(|\Phi\rangle) = -\frac{S(|\Phi\rangle)}{M} = 2\pi^2 \left(\frac{1}{2} \langle \Phi, Q_B \Phi \rangle + \frac{1}{3} \langle V_3 | |\Phi^{(1)}\rangle |\Phi^{(2)}\rangle |\Phi^{(3)}\rangle \right) \quad (18)$$

where the BRST operator is

$$Q_B = c_n L_{-n}^m + \frac{m-n}{2} : c_m c_n b_{-m-n} : - c_0, \quad (19)$$

while the three-string vertex $\langle V_3 |$ was defined in equation (5) and can be written as

$$\langle V_3 | = \frac{3^4 \sqrt{3}}{2^6} \sum_{i,j,k} \langle ij | \langle jk | \langle ki | c_{-1}^{(1)} c_{-1}^{(2)} c_{-1}^{(3)} c_0^{(1)} c_0^{(2)} c_0^{(3)} e^\Xi, \quad (20)$$

with

$$\Xi = \sum_{r,s=1}^3 \sum_{m,n=0}^{\infty} \left(\frac{1}{2} \alpha_m^{(r)\mu} N_{mn}^{rs} \alpha_{n,\mu}^{(s)} \right) + \sum_{r,s=1}^3 \sum_{m=1,n=0}^{\infty} (c_m^{(r)} X_{mn}^{rs} b_n^{(s)}) . \quad (21)$$

As usual, the r, s indices run from 1 to 3 and label the three strings interacting at the vertex.

To compute the quadratic terms in the potential, we use the properties of the bpz conjugate,

$$|ij\rangle \rightarrow \langle ij|, \quad \alpha_n^\mu \rightarrow (-1)^{n+1} \alpha_{-n}^\mu, \quad L_n \rightarrow (-1)^n L_{-n} \quad (22)$$

$$c_n \rightarrow (-1)^{n+1} c_{-n}, \quad b_n \rightarrow (-1)^n b_{-n}, \quad bpz(AB) = bpz(A)bpz(B)^3. \quad (23)$$

To compute the cubic terms in the potential, we need the three-string coefficients N_{mn}^{rs} and X_{mn}^{rs} . These are well known [19] and were originally computed for Neumann strings. T-duality guarantees that the same result applies to Dirichlet boundary conditions, at least as long as the D-brane coordinates are diagonal. The three-string coefficients we require are

³For purely commuting or purely anti-commuting A and B, $AB = \pm BA$ [18].

m n	N_{mn}^{rr}	$N_{mn}^{r(r+1)}$	$N_{mn}^{r(r-1)}$
0 0	$\ln 4\sqrt{3}/9$	0	0
0 1	0	$-2\sqrt{3}/9$	$2\sqrt{3}/9$
1 0	0	$2\sqrt{3}/9$	$-2\sqrt{3}/9$
0 2	$-2/27$	$1/27$	$1/27$
1 1	$-5/27$	$16/27$	$16/27$
2 0	$-2/27$	$1/27$	$1/27$
0 3	0	$22\sqrt{3}/729$	$-22\sqrt{3}/729$
1 2	0	$32\sqrt{3}/243$	$-32\sqrt{3}/243$
2 1	0	$-32\sqrt{3}/243$	$32\sqrt{3}/243$
3 0	0	$-22\sqrt{3}/729$	$22\sqrt{3}/729$

m n	X_{mn}^{rr}	$X_{mn}^{r(r+1)}$	$X_{mn}^{r(r-1)}$
1 0	0	$4\sqrt{3}/9$	$-4\sqrt{3}/9$
1 1	$11/27$	$8/27$	$8/27$
2 0	$16/27$	$-8/27$	$-8/27$
1 2	0	$40\sqrt{3}/243$	$-40\sqrt{3}/243$
2 1	0	$-80\sqrt{3}/243$	$80\sqrt{3}/243$
3 0	0	$-68\sqrt{3}/243$	$68\sqrt{3}/243$

Table 3: The first few Neumann coefficients appearing in equation (21). The matter coefficients are taken from [20] and the ghost coefficients from [19]. The non-zero mode matter coefficients and the ghost coefficients agree with equations (141) and (147) in [18].

given in Table 3. It will be useful later to define $\tilde{N}_{mn}^{rs} = N_{mn}^{rs} + N_{nm}^{sr}$. Given the string field Φ , it is straight-forward to compute $e^{\Xi}\Phi e^{-\Xi}$. Using the fact that

$$e^{\Xi}|ij\rangle|jk\rangle|ki\rangle = \left(\frac{4}{3\sqrt{3}}\right)^{\frac{1}{2}((d_{ij})^2+(d_{jk})^2+(d_{ki})^2)}|ij\rangle|jk\rangle|ki\rangle, \quad (24)$$

the coefficients $g(d_{ij}, d_{jk}, d_{ki})$ defined in equation (6) can then be obtained using a computer-assisted algebra program.

For example, at level (1,3), we obtain

$$\begin{aligned} f(|\Phi\rangle) = & 2\pi^2 \left[\frac{1}{2} \sum_{ij} \left(\left(-1 + \frac{(d_{ij})^2}{2} \right) t_{ij} t_{ji} + \frac{(d_{ij})^2}{2} x_{ij} x_{ji} - 2h_{ij} h_{ji} \right) \right. \\ & \left. + \frac{3^3 \sqrt{3}}{2^6} \sum_{ijk} \left(t_{ij} t_{jk} t_{ki} + \frac{3}{2} \tilde{N}_{01}^{r3} \alpha_0^{(r)} t_{ij} t_{jk} x_{ki} + \frac{3}{2} \left(\tilde{N}_{11}^{32} + \frac{1}{2} \tilde{N}_{01}^{r2} \tilde{N}_{01}^{s3} \alpha_0^{(r)} \alpha_0^{(s)} \right) t_{ij} x_{jk} x_{ki} \right) \right] \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{4} \left(\tilde{N}_{11}^{12} \tilde{N}_{10}^{3r} \alpha_0^{(r)} + \tilde{N}_{11}^{23} \tilde{N}_{10}^{1r} \alpha_0^{(r)} + \tilde{N}_{11}^{31} \tilde{N}_{10}^{2r} \alpha_0^{(r)} + \frac{1}{2} \tilde{N}_{01}^{r1} \tilde{N}_{01}^{s2} \tilde{N}_{01}^{t3} \alpha_0^{(r)} \alpha_0^{(s)} \alpha_0^{(t)} \right) x_{ij} x_{jk} x_{ki} \\
& + \frac{16}{9} t_{ij} h_{jk} h_{ki} + \frac{8}{9} \tilde{N}_{01}^{r3} \alpha_0^{(r)} h_{ij} h_{jk} x_{ki} \left(\frac{4}{3\sqrt{3}} \right)^{\frac{1}{2}((d_{ij})^2 + (d_{jk})^2 + (d_{ki})^2)} \Big] \quad (25)
\end{aligned}$$

where we have omitted the index 25 on the α_0 oscillators for clarity and where $\alpha_0^{(1)} \rightarrow d_{ij}$, $\alpha_0^{(2)} \rightarrow d_{jk}$ and $\alpha_0^{(3)} \rightarrow d_{ki}$.

2.4 Two D-branes and exchange symmetry

The results up to this point were applicable to any number N of parallel D24-branes. Specializing to $N = 2$, we define $d = d_{21}$, so that $\alpha_0^{25}|21\rangle = d|21\rangle$ and $\alpha_0^{25}|12\rangle = -d|12\rangle$. d is proportional to the distance between the two D-branes.

A useful parametrization of the matrix-valued fields (assuming a twist-even string field) is

$$t_{ij} = \begin{bmatrix} T_s - T_a & \tau \\ \tau & T_s + T_a \end{bmatrix} \quad x_{ij} = \begin{bmatrix} X_s - X_a & \chi \\ \chi & X_s + X_a \end{bmatrix} \quad (26)$$

and similar for all the real (and therefore symmetric) fields, and

$$h_{ij} = \begin{bmatrix} 0 & \gamma \\ -\gamma & 0 \end{bmatrix} \quad (27)$$

and similar for all the imaginary fields.

At level (1,3) the potential is given explicitly by

$$\begin{aligned}
f(T_s, T_a, \tau, X_s, X_a, \chi, \gamma) = & 2\pi^2 \left[-T_s^2 - T_a^2 + \left(-1 + \frac{d^2}{2} \right) \tau^2 + \frac{d^2}{2} \chi^2 + 2\gamma^2 \right. \\
& + \frac{27\sqrt{3}}{32} T_s^3 + \frac{81\sqrt{3}}{32} T_s T_a^2 + \frac{3\sqrt{3}}{2} T_s (X_s^2 + X_a^2) + 3\sqrt{3} T_a (X_s X_a) \\
& + \left(\frac{4}{3\sqrt{3}} \right)^{d^2} \left(\frac{81\sqrt{3}}{32} T_s \tau^2 + \frac{27d}{8} X_a \tau^2 - \frac{27d}{8} T_a \tau \chi + 3\sqrt{3} \left(1 - \frac{d^2}{2} \right) X_s \tau \chi \right. \\
& \left. \left. + \frac{3\sqrt{3}}{2} \left(1 + \frac{d^2}{4} \right) T_s \chi^2 + \frac{d^3}{2} X_a \chi^2 - \frac{3\sqrt{3}}{2} T_s \gamma^2 - 2d X_a \gamma^2 \right) \right] \quad (28)
\end{aligned}$$

When working with just two D-branes, there is an extra symmetry we can take advantage of. Since we are not interested in the marginal deformation which moves both D-branes together in a rigid way, we will confine ourselves to those solutions which correspond to D-brane configurations symmetric under $X^{25} \rightarrow -X^{25}$. The action is invariant under simultaneously taking $X^{25} \rightarrow -X^{25}$ (or, equivalently, $\alpha_n^{25} \rightarrow -\alpha_n^{25}$ for all n) and $1 \leftrightarrow 2$. We will refer to this as the exchange symmetry. If we are interested in solutions where the tachyon

field does not break this symmetry, we can restrict ourselves to exchange symmetry even fields (as exchange symmetry odd fields cannot appear linearly in the action). This means that for fields with an even number of α_{-n}^{25} in them, we can set $A_{11} - A_{22}$ and $A_{12} - A_{21}$ to zero (this, at level one, implies that $T_a = 0$ and $\gamma = 0$) and for fields with an odd number of α_{-n}^{25} , we can set $A_{11} + A_{22}$ and $A_{12} + A_{21}$ to zero (at level one, $X_s = 0$ and $\chi = 0$). It is easy to see from the potential in equation (28) that we obtain a consistent truncation of the theory at this level. Truncating to exchange even fields allows us to drop h , o_1 , o_2 , p and q altogether. With this truncation, we can compute the complete cubic action at level (3,9). The quadratic part of the potential up to level 3 can be found in the Appendix, while the cubic couplings including fields up to at level 3 will be published separately [21]. Where possible, the coefficients have been verified to agree with those previously computed, for example in [5].

3 D-brane decay

3.1 Level 0

We begin to analyze D-brane decay for two separated D-branes at level 0, where only the tachyon fields come into play and where the solutions can be obtained analytically. The potential, without restricting to exchange even fields, is

$$f(T_s, T_a, \tau) = 2\pi^2 \left[-T_s^2 - T_a^2 + \left(-1 + \frac{d^2}{2} \right) \tau^2 + \frac{81\sqrt{3}}{32} \left(\frac{1}{3} T_s^3 + T_s T_a^2 + \left(\frac{4}{3\sqrt{3}} \right)^{d^2} T_s \tau^2 \right) \right]. \quad (29)$$

There are five points where the derivatives of the potential (29) vanish. These include the four expected solutions: the perturbative vacuum at $T_s = T_a = \tau = 0$, two solutions corresponding to one of the two D-branes decaying ($T_s = \pm T_a = T_0/2$, $\tau=0$), and one solution corresponding to both D-branes decaying, ($T_s = T_0$, $T_a = 0$, $\tau = 0$), where $T_0 = 64/81\sqrt{3}$ is the well-know level 0 approximation to the tachyon field (see for example [1][18]). In addition, there is a new, non-diagonal, exchange-even solution. The energy and tachyon fields for this solution are shown in Figure 1. At zero separation, this solution corresponds simply to single D-brane tachyon condensation, though the D-brane which undergoes decay is an $SU(2)$ rotation of our two original D-branes:

$$\begin{bmatrix} T_s - T_a & \tau \\ \tau & T_s + T_a \end{bmatrix} = \begin{bmatrix} T_0/2 & T_0/2 \\ T_0/2 & T_0/2 \end{bmatrix} = U^\dagger \begin{bmatrix} T_0 & 0 \\ 0 & 0 \end{bmatrix} U, \quad (30)$$

where $U = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$.

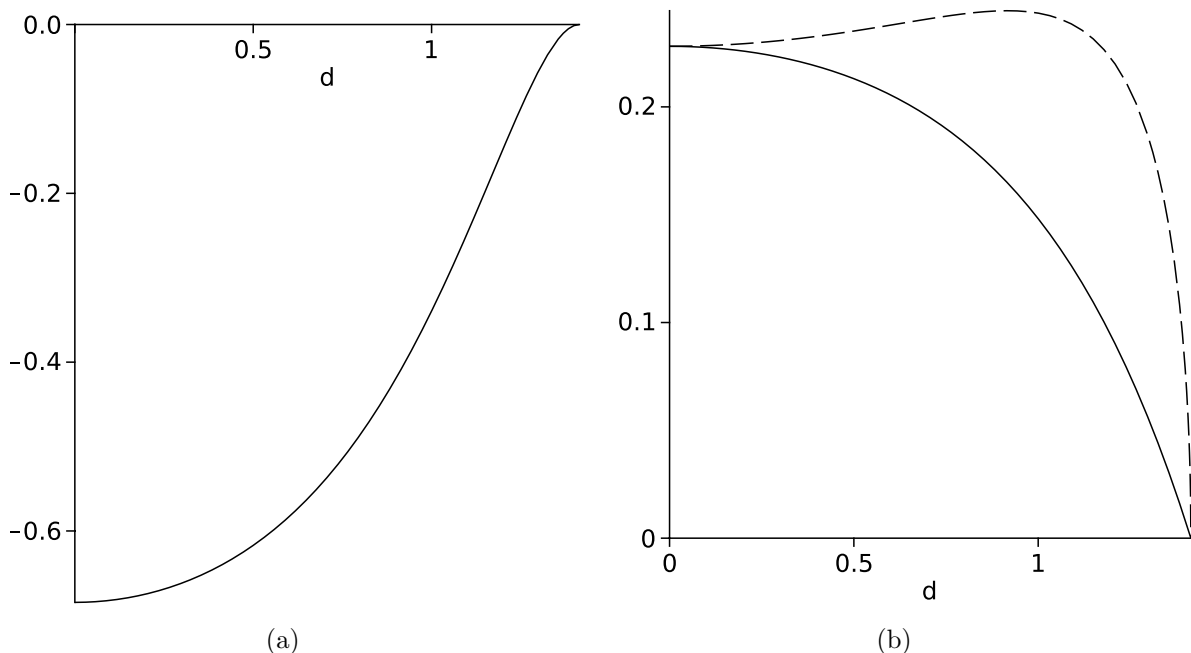


Figure 1: The non-diagonal solution at level 0. (a) Energy as a function of D-brane separation, d . (b) The tachyon field: T_s (solid line) and τ (dashed line) as a function of D-brane separation ($T_a = 0$).

The persistence of this solution for separated D-branes, away from the $SU(2)$ symmetric point, is at first surprising. Since it seems unlikely that this solution somehow corresponds to each of the two D-branes having decayed ‘half-way’, we propose the following interpretation: the solution corresponds to the two separated D-branes moving towards each other until coincident, restoring $SU(2)$ symmetry before the decay occurs. The solution shown in Figure 1 should then correspond to a combination of D-brane translation (a marginal deformation) and tachyon decay. To test this hypothesis, we analyze this solution at higher truncation levels. Since the solution is exchange-even at level 0, we focus only on the exchange-even sector, as described in Section 2.4.

3.2 Level (1,3)

When we impose the exchange symmetry, the potential is quite simple and involves only T_s , τ and X_a :

$$f(T_s, \tau, X_a) = 2\pi^2 \left[-T_s^2 + \left(-1 + \frac{d^2}{2} \right) \tau^2 + \frac{27\sqrt{3}}{32} T_s^3 + \frac{3\sqrt{3}}{2} T_s X_a^2 + \right] \quad (31)$$

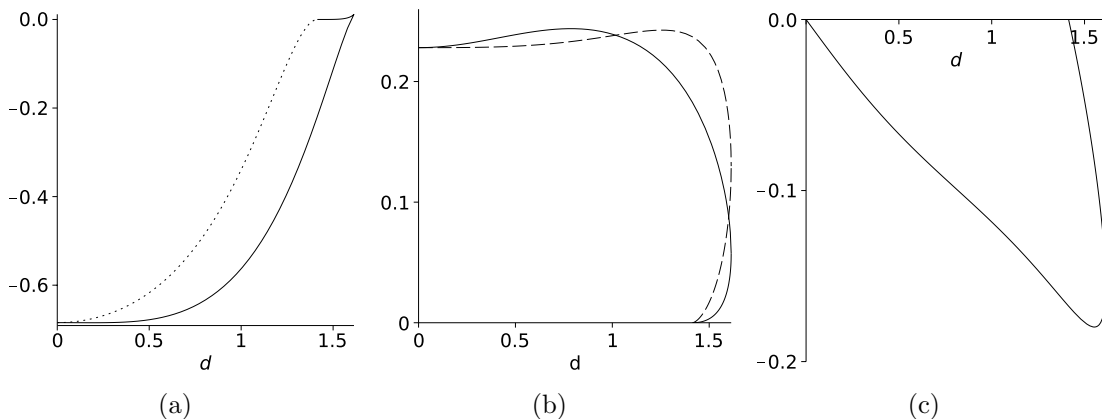


Figure 2: The non-diagonal solution at level (1,3), as a function of initial D-brane separation, d . (a) Energy (the dashed line shows the energy at level 0, from Figure 1a, for comparison). (b) The tachyon field: T_s (solid line) and τ (dashed line). (c) The transverse scalar field X_a .

$$+ \left(\frac{4}{3\sqrt{3}} \right)^{d^2} \left(\frac{81\sqrt{3}}{32} T_s \tau^2 + \frac{27d}{8} X_a \tau^2 \right) \Bigg] ,$$

At this level, the string field includes the transverse scalar field X_a , which will play a central point in the remainder of this paper. This field is the infinitesimal marginal parameter for a D-brane translation mode moving the two D-branes symmetrically either further apart or closer together [3].

The non-diagonal solution of interest at this level is presented in Figure 2. It can be found analytically.

Let us first discuss several properties of this solution at relatively small values of D-brane separation.

- X_a is negative and approximately linear as a function of separation. The small d behaviour in Figure 2c implies that the D-branes are moving closer together by an amount proportional to the initial separation. If, as we argued, this amount is in fact equal to the initial separation (so that SU(2) symmetry is restored), Figure 2c can be interpreted as showing the relationship between the parameter X_a and the physical distance by which the D-brane has moved in our solution. We will return to this point in section 3.4.
- The energy as a function of separation is much flatter for level 1 than for level 0. This again supports our hypothesis: if the solution we are studying corresponds to the two D-branes coming together by a marginal deformation followed by a decay of some combination of the two coincident D-branes once SU(2) symmetry has been

restored, the energy released in the decay should be the same no matter what the original separation was.

- As we already discussed, at zero separation, the solution has $T_s = \tau = T_0/2$, which is equivalent under a $SU(2)$ conjugation to $T_s \pm T_a = T_0$, $T_s \mp T_a = \tau = 0$, the solution corresponding to the decay of either the left or the right brane. As we see in Figure 2b, $T_s \approx \tau \approx T_0/2$ up to $d \approx 1.2$. We take this to indicate that at this level, the contribution to T_s due to nonzero marginal deformation parameter X_a is quite small. As we will see, this is not so at higher levels.

Behaviour at larger separations is also quite interesting: at separation $d = \sqrt{2}$, a new solution appears, and eventually merges with our original solution around $d \sim 1.6$ (this is why the plots in Figure 2 are double valued in this range). $d = \sqrt{2}$ is the point where the off-diagonal tachyon string becomes massless. It is not surprising that an appearance of a nearly massless mode results in a new branch of the solution. That this branch merges with our solution of interest is somewhat similar to what was found in [3]. (This point will also be discussed in more detail in Section 3.4.) No solutions can be found beyond the point where the two branches merge. We also note that the branch which we interpret to represent tachyon decay exists past the point where the off-diagonal tachyon string becomes massless. This further reinforces our interpretation of this branch: once the two D-branes have come together, the off-diagonal tachyon is tachyonic once more.

3.3 Level (3,9)

At level 3, in addition to t and x , we have the following twist-even, exchange-even fields:

$$\begin{aligned} u_{ij} &= \begin{pmatrix} U_s & v \\ v & U_s \end{pmatrix}, & v_{ij} &= \begin{pmatrix} V_s & \nu \\ \nu & V_s \end{pmatrix}, & \tilde{w}_{ij} &= \begin{pmatrix} \tilde{W}_s & \tilde{\omega} \\ \tilde{\omega} & \tilde{W}_s \end{pmatrix}, & \tilde{f}_{ij} &= \begin{pmatrix} 0 & \tilde{\phi} \\ -\tilde{\phi} & 0 \end{pmatrix}, \\ r_{ij} &= \begin{pmatrix} -R_a & 0 \\ 0 & R_a \end{pmatrix}, & s_{ij} &= \begin{pmatrix} -S_a & 0 \\ 0 & S_a \end{pmatrix}, & y_{ij} &= \begin{pmatrix} -Y_a & 0 \\ 0 & Y_a \end{pmatrix}, & z_{ij} &= \begin{pmatrix} -Z_a & 0 \\ 0 & Z_a \end{pmatrix}. \end{aligned} \quad (32)$$

We are using \tilde{w} and \tilde{f} instead of w and f (see Table 2) because, as explained in Section 2.1, w and f are singular at $d = \sqrt{2}$. Figures 3 and 4 show these fields, as well as the energy of the solution, as functions of the initial D-brane separation, d .

3.4 Discussion

We would like to interpret the solution we have found as representing a combination of a marginal deformation bringing the two D-branes to the same position and a tachyon condensation diagonal in some new basis. This new basis is an $SU(2)$ rotation of the original basis, possible because $SU(2)$ symmetry has been restored by the marginal deformation. One

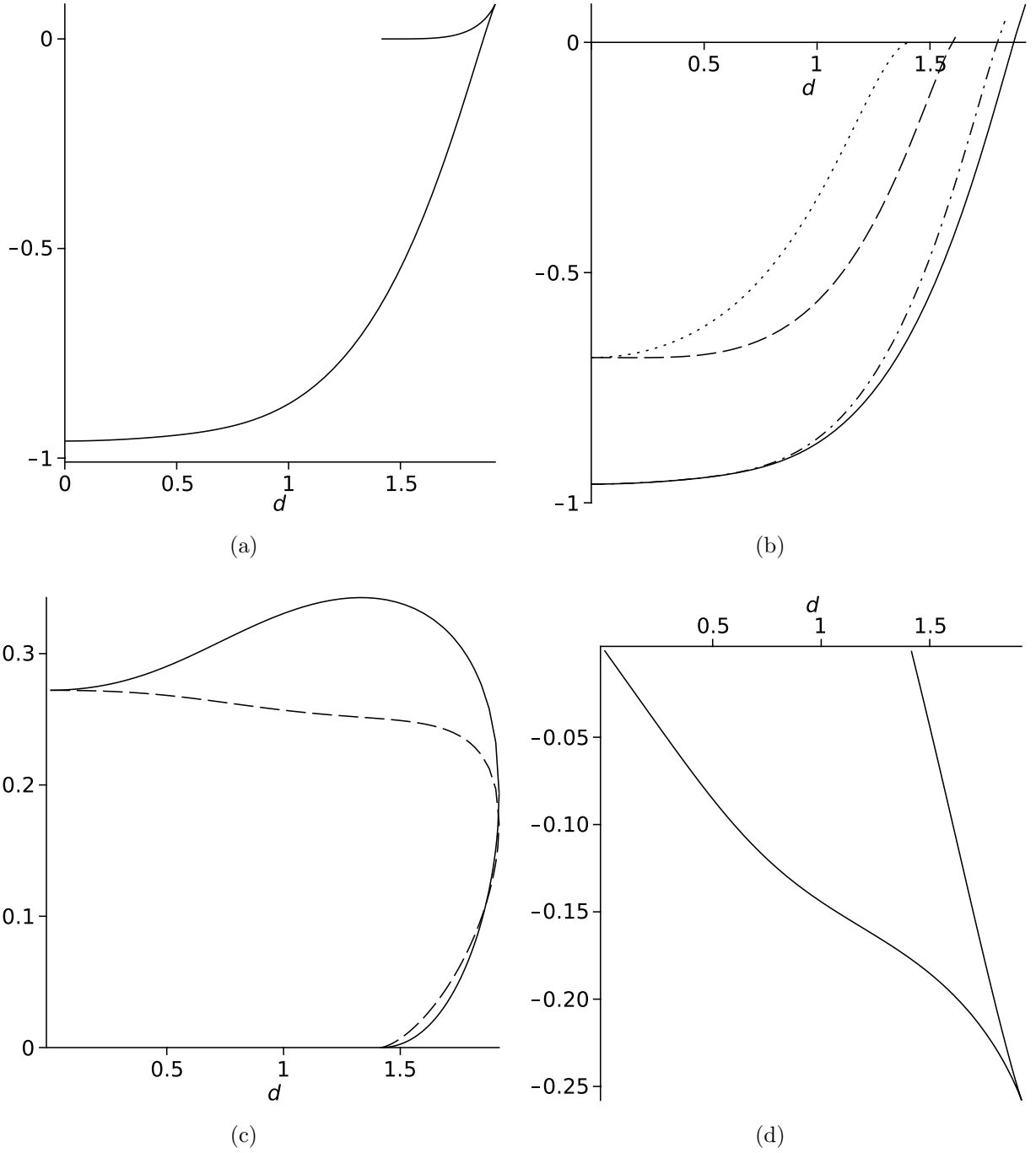


Figure 3: The non-diagonal solution at level (3,9), as a function of initial D-brane separation, d . (a) Energy. (b) Energy at level 3 (solid line), level 2 (dash-dot), level 1 (dash) and level 0 (dotted line), for comparison. For clarity, only the first branch is shown. (c) The tachyon field: T_s (solid line) and τ (dashed line) (d) The transverse scalar field X_a .

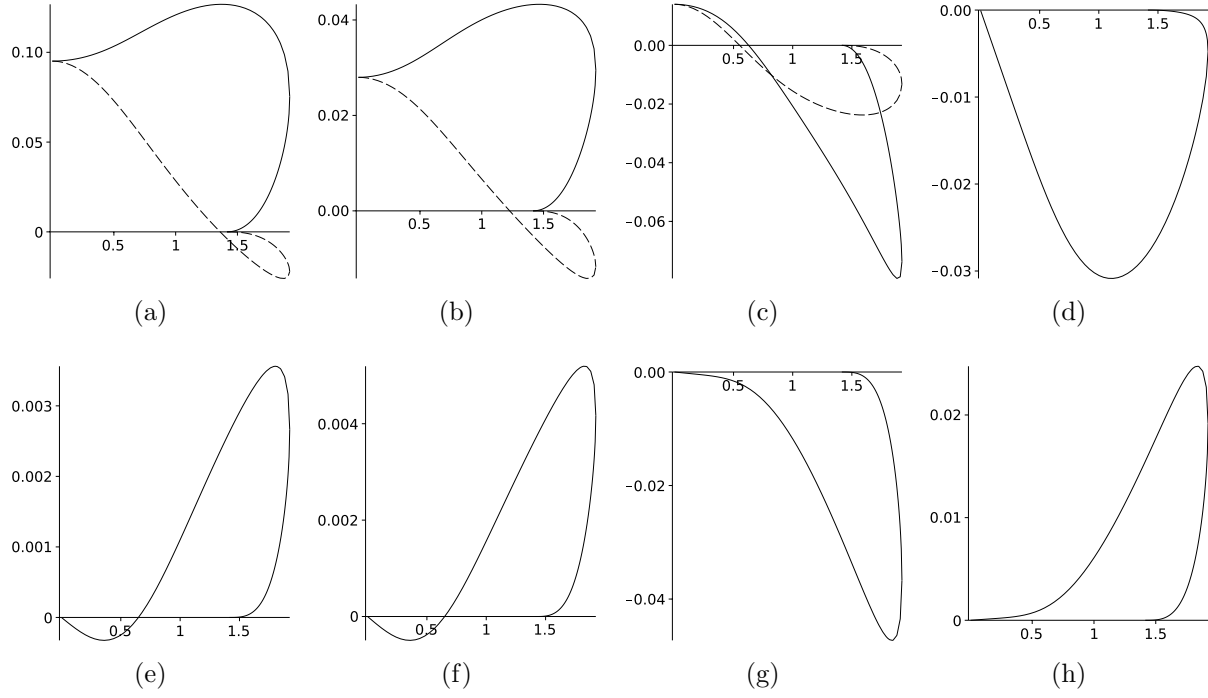


Figure 4: The other fields at level (3,9), as a function of initial D-brane separation, d . (a) U_s (solid line) and v (dashed line). (b) V_s (solid line) and ν (dashed line). (c) \tilde{W}_s (solid line) and \tilde{w} (dashed line). (d) $\tilde{\phi}$. (e) R_a . (f) S_a . (g) Y_a . (h) Z_a .

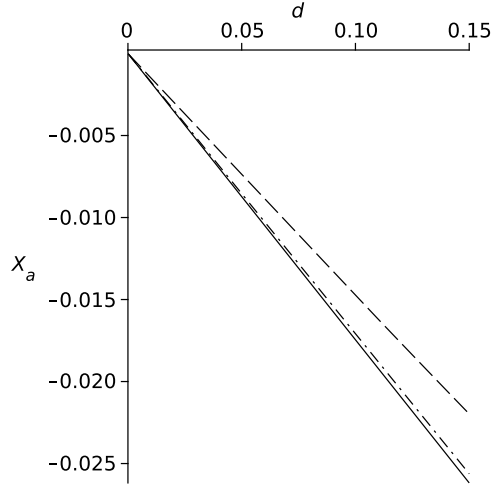


Figure 5: The field X_a as a function of the rescaled D-brane separation d for small separations at different truncation levels. Level (3,9) is solid line and lower levels (2 and 1) have shallower slopes.

should ask then why we only see one such solution (especially at level (1,3), where we have not imposed exchange symmetry and where the full $SU(2)$ family should be visible). The answer is that level truncation does not allow for a full restoration of the $SU(2)$ symmetry, and only solutions at isolated points survive. To see that level truncation affects the restoration of $SU(2)$ consider two separated D-branes in the full theory. There exists a solution in this theory corresponding to bringing the two D-branes together. Expanding the potential around this solution and performing a field redefinition⁴, we should get back the $SU(2)$ symmetric potential for fields living on two coincident D-branes. The field redefinition mixes fields at different levels and is not the same for diagonal and off-diagonal elements, because X_a is nonzero while χ is not. Fully restoring $SU(2)$ symmetry is therefore not possible in the truncated theory.

Several small comments on Figure 3 are in order. There are two branches visible, one starting at zero separation, $d = 0$, and the second one starting at $d = \sqrt{2}$ (the second branch starts at this point independent of the level of truncation). The two branches merge and end at $d \approx 1.92$ (beyond this point, the solutions become complex). We attribute the existence of the second branch to the fact that at $d = \sqrt{2}$, the off-diagonal element of the tachyon field is massless. The shape of the $X_a(d)$ curve (Figure 3d) near the point where the two branches meet cannot be determined at this level of truncation: it might be a cusp or possibly even a loop. In Figure 3c we see that T_s is no longer approximately equal to τ . This is due to the the marginal deformation component of the solution which has $T_s \neq \tau = 0$. Figure 3b shows the energy of the solution at different truncation levels. Surprisingly, the energy is somewhat less flat at levels 2 and 3 than it is at level 1. The decrease in flatness when going from level 1 to level 2 might be related to the observation in [3] that the leading quadratic term in the vacuum branch of the effective potential for a_s , analogous to our field X_a , is larger at level 2 than at levels 1 or 3. Because the marginal direction is lifted by level truncation, we expect that the curves would become flatter again if the truncation level were increased, despite the increased curvature when we go from level 1 to level 2.

It is tempting to interpret the curves in Figure 3d and in Figure 5 as corresponding to the relationship between the vev of the marginal parameter in SFT, X_a , and the physical displacement of the D-branes, d . Unfortunately, this would not be correct, even at linear level at small d , as seen in Figure 5. To see why, consider the general form of the SFT (untruncated) potential with D-brane separation d , $f_d(X_a, \varphi^I, \xi^I)$, where we have split the fields in this potential into three groups: the massless mode X_a , all the other diagonal fields φ^I and the off-diagonal fields ξ^I . For X_a to be massless, the potential must contain no X_a^2 term and no terms of the form $X_a \varphi_I$ or $X_a \xi_I$. Further, no term can be linear or cubic in the fields ξ^I . On the other hand, the potential must contain terms of the form $X_a^2 \phi^I$ whose coefficients do not depend on d . Let $X_a = \bar{X}_a \neq 0$, $\varphi^I = \bar{\varphi}^I$, $\xi^I = 0$ be a

⁴For infinitesimal marginal deformations, these field redefinitions were computed in [22].

solution representing D-brane translation such that the two D-branes are coincident. For small initial D-brane separations, \bar{X}_a is simply proportional to d , but for larger separations their relationship is more complicated. Expanding around this solution, $X_a = \bar{X}_a + X'_a$, $\varphi^I = \bar{\varphi}^I + \varphi'^I$, $\xi^I = \xi'^I$ we obtain a potential for the new fields X'_a , φ'^I and ξ'^I . With an appropriate field redefinition, $(X'_a, \varphi'^I, \xi'^I) \rightarrow (\tilde{X}_a, \tilde{\varphi}^I, \tilde{\xi}^I)$, this potential is equal to the potential at zero separation, f_0 :

$$f_d(\bar{X}_a + X'_a, \bar{\varphi}^I + \varphi'^I, \xi'^I) = f_0(\tilde{X}_a, \tilde{\varphi}^I, \tilde{\xi}^I) \quad (33)$$

and the SU(2) symmetry is apparent. We will work to leading order, where the field redefinition is linear. Thus, a particular linear combination of X'_a and φ'^I , $\tilde{X}_a \approx c_{XX}X'_a + \sum_I c_{XI}\varphi'^I$, appears massless in f' (meaning that when written in terms of the redefined fields, f' has no term quadratic in \tilde{X}_a). Considering explicitly the expansion of the potential $f_d(\bar{X}_a + X'_a, \bar{\varphi}^I + \varphi'^I, \xi'^I)$ we see that it must contain terms of the form $\bar{X}_a X'_a \varphi'_j$, with some d -independent coefficients. At small separations, \bar{X}_a is proportional to d and so this cross-term between X'_a and φ'_j has a coefficient proportional to d . This leads to mixing between X'_a and φ'_j in the new massless eigenfield, \tilde{X}_a . Explicitly, as the separation between D-branes goes to zero, $d \rightarrow 0$, $c_{XX} \rightarrow 1$ and $c_{XI} \sim d \rightarrow 0$. Similarly, for $\tilde{\varphi}^I \approx c_{IX}X'_a + \sum_J c_{IJ}\varphi'^J$, we have that $c_{II} \rightarrow 1$ while $c_{IX}, c_{IJ} \rightarrow 0$ (for $I \neq J$).

In the new ‘tilde’ variables, there is a SU(2) family of solutions representing the decay of a single D-brane. These solution have $\tilde{X}_a = 0$, while $\tilde{\varphi}^I$ are nonzero and do not depend on the initial D-brane separation. Thus, X'_a is non-zero, unless there is some cancellation, which we have no reason to expect. More explicitly, at small d , we have that $\varphi'^J \approx \tilde{\varphi}^J$, and $0 = \tilde{X}_a \approx X'_a + \sum_I c_{XI}\varphi'^I$ so that $X'_a \approx -\sum_I c_{XI}\tilde{\varphi}^I$. Since we already argued that c_{XI} decrease linearly with $d \rightarrow 0$, this implies that X'_a is also linear in d . Therefore, in the combined translation-and-decay solution, the vev of the massless mode is $X_a = \bar{X}_a + X'_a$, is not the same as it would be with translation alone. The correction, X'_a , is of the same order in d as \bar{X}_a itself, so the vertical axis in Figure 3d does not represent the vev of the marginal parameter responsible for a translation, even at small d . To be able to understand in detail the relationship between the vev of the massless SFT field and the vev in the CFT from our solutions, we would need to understand the field redefinition between φ' and $\tilde{\varphi}$. We leave this problem for future work, but point out that solving it requires only a better understanding of the marginal deformation in SFT and not of any connections between SFT and the CFT.

Still, because it was computed by controlling the D-brane displacement itself as the adjustable parameter, and not a parameter in the string field, Figure 3d contains a very interesting piece of information: There is a finite maximum D-brane separation $d \approx 1.9$ beyond which the solutions do not exist. This corresponds to a physical separation between the D-branes equal to $\pi\sqrt{2\alpha'}d \approx 8.5\sqrt{\alpha'}$. The implication is that open string field theory in this particular coordinate system is unable to describe the displacement of a D-brane beyond

half this distance, and therefore fails to describe the full CFT moduli space.

This answers the question raised in [3]. In that paper, a marginal deformation is studied by assuming a predetermined value for a certain marginal parameter in the string field (a_s in [3], T-dual to our X_a), solving the equations of motion of all *other* fields and thus computing the effective potential for a_s . Level truncation lifts this potential and what is obtained is not truly a solution to the complete string field theory equations of motion, as the equation of motion for a_s is not satisfied. It is found there that even the equations of motion for the remaining fields cannot be solved at all once a_s is greater than some critical value \bar{a}_s , and that at $a_s = \bar{a}_s$ the ‘solution’ in merges with another branch. The value of \bar{a}_s computed in [3] is about 0.331 (at level (4,8)). In contrast, we find *actual* solutions to the truncated equations of motion, but our solution is a combination of the marginal deformation and a decay of one of the two D-branes. We also find that there are no solutions beyond a certain point; the largest $|X_a|$ attained for our solutions is 0.1557 (level 1), 0.2431 (level 2) and 0.2579 (level 3). As we have already discussed, this is not the actual marginal parameter, so we cannot compare our values with those of [3], though we note they are of the same order of magnitude. Qualitatively we do see the same phenomenon: the marginal deformation has a finite range. However, since we have the physical distance through which the D-branes have been displaced, we can also say that this finite range of marginal deformation parameter corresponds to a finite range of the CFT vev, which the authors of [3] were unable to do.

It is interesting to ask whether the finite range of deformation can be an artifact of either truncation or breakdown of Siegel gauge. In Figure 3b we see that there is some indication of convergence with increased level, including convergence of the range over which the deformation exists. It would be quite interesting to explore the truncation at higher levels. In particular, it might be that increasing the level from even to odd has a smaller effect than increasing the level from odd to even. If that is the case, a computation at level 4 could be quite telling. The question about gauge validity is hard to settle given our data. While the value of tachyon field is well within the region where Siegel gauge holds [23, 24] for the entire solution including the branch point, (Figure 3(c)), it is possible that the breakdown of Siegel gauge occurs at a smaller tachyon field in our set up. It would be interesting to explore this possibility, by computing the effective potential for the tachyon at different brane separations and repeating the analysis of [24] for multiple separated D-branes.

An interesting interpretation of the results in [3] was given in [25]: there it was proposed that there is another branch of solutions to the SFT equations of motion, so that, at the same value of the marginal deformation parameter there can be two solutions, differing in the higher level fields, representing two different vevs in the CFT. This would allow OSFT to cover the full CFT moduli space. We find no indication in our computation of the existence of such a branch.

4 Restoration of SU(2) symmetry

In the previous section, we used the presence of an off-diagonal tachyon condensate as a signal that the two D-branes have been brought together and SU(2) symmetry has been restored. In this section, we discuss an attempt to find solutions corresponding to the SU(2) symmetric point directly, by examining the spectrum of the theory around an approximate solution corresponding to a purely marginal deformation.

Starting with two spatially separated D-branes, there should exist a solution in the untruncated SFT which corresponds to bringing these two D-branes together, restoring SU(2) symmetry. The SFT action for small fluctuations around such a string field will have an explicit SU(2) symmetry, reflected, for example, in the degeneracies of the mass spectrum for these small fluctuations. Unfortunately, due to mixing of fields at different levels, in a level-truncated model the symmetry is restored only approximately.

Our strategy is as follows: starting with a theory with a given D-brane separation d , we construct a one parameter set of approximate solutions corresponding to different marginal deformations bringing the two D-branes closer together by a varying amount. These approximate solutions are constructed using the approach in [3], *i.e.* by picking a marginal deformation parameter X_a and solving the equations of motion for all the other fields in the potential. Once we have the approximate solution, we find the matrix of second derivatives of the potential w.r.t. all the fields and diagonalize it. If we were to perform this computation in the untruncated theory, we would expect to find that at a particular value of X_a , $\bar{X}_a(d)$, the spectrum would develop degeneracies at the point where the SU(2) symmetry is restored. In addition, near the degenerate point, the spectrum as a function of the X_a should be reflection-symmetric about $X_a = \bar{X}_a(d)$ —bringing the D-branes nearly together should produce the same spectrum as ‘overshooting’ a bit. Identifying the degenerate point for different values of d would produce the function $\bar{X}_a(d)$, describing the relationship between the strength of the OSFT deformation \bar{X}_a and the CFT vev it produces, $d/2$.

In a level truncated theory, we would hope that this degeneracy is present at least approximately. Note that while we focus on the matrix of second derivatives, in level truncation, the first derivatives of the potential do not all vanish, since the equation of motion for X_a is not satisfied. This effect decreases with increased truncation level.

Our results, for the eigenvalues corresponding to the masses of selected fields are shown in Figure 6. The computation used twist-even fields only. The approximate solution is also exchange-even, and we included all twist-even (both exchange-even and exchange-odd) fields in the computation of the second derivative matrix. At zero separation, the fields t and u (6a,6c) have masses equal -1 while the fields in s have mass 25 (6b,6d) (see Appendix). Unfortunately, the features we just discussed do not seem to be unambiguously visible at level (3,9). Apparently, the cubic couplings to higher level fields with non-zero vev contribute nontrivially to the masses of the lower modes when the D-branes are translated. It would

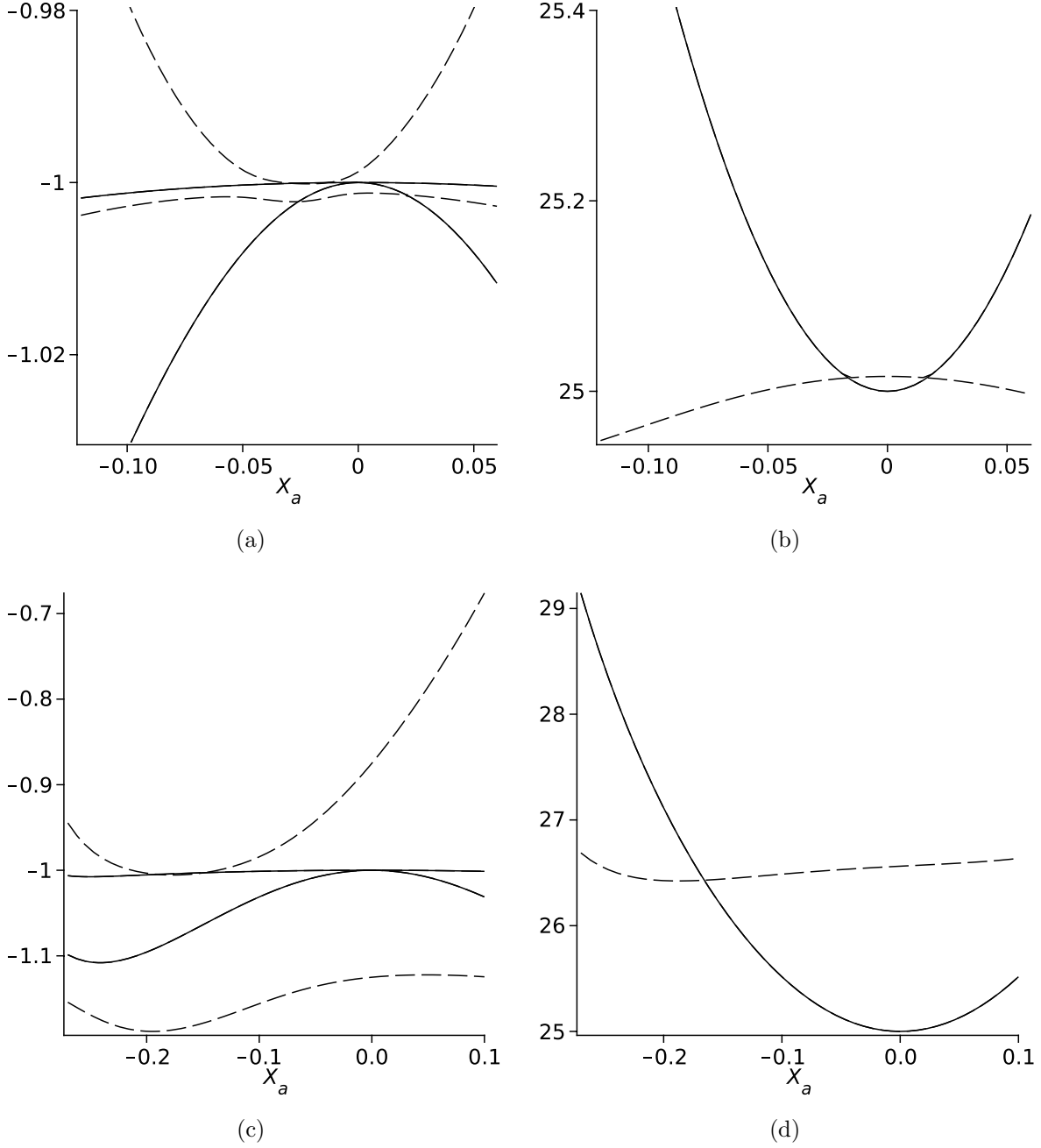


Figure 6: Eigenvalues of the matrix of second derivatives of the potential near an approximate solution for a fixed X_a , as a function of X_a . The initial D-brane separation corresponds to $d = 0.05$ in (a) and (b) and to $d = 0.5$ in (c) and (d). (a),(c) Eigenvalues corresponding to the fields t_{ij} and u_{ij} . (b),(d) Eigenvalues corresponding to the fields s_{ij} . Dashed lines indicate a non-degenerate eigenvalue while solid lines correspond to doubly-degenerate eigenvalues.

be interesting to see whether this can be improved at higher levels.

Appendix: The potential at higher levels

Let us denote our set of string fields, t, x, u, \dots , with $\phi^{(m)}$. Then the string field potential (18) can be written as

$$\begin{aligned} f &= \pi^2 \sum_{l,m} \sum_{ij} A_{lm}(d_{ij}) \phi_{ij}^{(l)} \phi_{ji}^{(m)} \\ &- 2\pi^2 \sum_{l,m,n} \sum_{ijk} B_{lmn}(d_{ij}, d_{jk}, d_{ki}) \left(\frac{4}{3\sqrt{3}} \right)^{\frac{1}{2}(d_{ij}^2 + d_{jk}^2 + d_{ki}^2)} \phi_{ij}^{(l)} \phi_{jk}^{(m)} \phi_{ki}^{(n)}, \end{aligned} \quad (34)$$

From equation (3) we get that $A_{lm} = \beta_m \beta_l A_{ml}$. The symmetry properties of the coefficients $B_{lmn}(d_{ij}, d_{jk}, d_{ki})$ were discussed in section 2.1 (where these coefficients were denoted with $g(d_{ij}, d_{jk}, d_{ki})$). Notice that the parameters d_{ab} appearing in the coefficients $A_{lm}(d_{ij})$ and $B_{lmn}(d_{ij}, d_{jk}, d_{ki})$ are just the eigenvalues of α_0^{25} in the lowest state of each ab sector of our theory. These can have a parallel interpretation in other scenarios, such as string theory on a single D-brane on a circle. This allows us to compare some of our coefficients to those computed for example in [5].

The coefficients $A_{lm}(d)$ for the quadratic part of the potential appear in Table 4. The string fields o and \tilde{o} are defined by $o_1 = o + \tilde{o}$ and $o_2 = 2\tilde{o} - 2o$. The coefficients $B_{lmn}(d_1, d_2, d_3)$ for the cubic part of the potential up to level (3,9) have the form of polynomials in (d_1, d_2, d_3) for example:

$$\begin{aligned} B_{xxw}(d_1, d_2, d_3) &= -\frac{1}{864} \sqrt{3} (4 d_1 d_2^3 - 108 + 12 d_3^3 d_2 + 12 d_3^3 d_1 - 37 d_1 d_2 - 24 d_3^2 d_2 d_1 \\ &- 8 d_1^2 d_2^2 + 4 d_2 d_1^3 - 4 d_3 d_2^3 + 155 d_3^2 + 16 d_1^2 + 16 d_2^2 + 8 d_3 d_2 d_1^2 \\ &- 8 d_3^4 - 75 d_3 d_1 - 75 d_3 d_2 - 4 d_3 d_1^3 + 8 d_3 d_1 d_2^2). \end{aligned} \quad (35)$$

A full set of these coefficients will appear elsewhere [21].

Acknowledgments

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$l\ m$	$A_{lm}(d)$
$t\ t$	$1/2\ d^2 - 1$
$x\ x$	$1/2\ d^2$
$h\ h$	-2
$u\ u$	$-1 - 1/2\ d^2$
$v\ v$	$\frac{25}{2} + \frac{25}{4}\ d^2$
$w\ w$	$1/4\ (4\ d^2 + 1)(d^2 + 2)$
$w\ f$	$3/2\ d(d^2 + 2)$
$f\ w$	$-3/2\ d(d^2 + 2)$
$f\ f$	$-(d^2 + 2)(d^2 + 1)$
$o\ o$	$8 + 2\ d^2$
$\tilde{o}\ \tilde{o}$	$-8 - 2\ d^2$
$p\ p$	$-100 - 25\ d^2$
$q\ q$	$-1/2\ (3\ d^2 + 2)(4 + d^2)$
$q\ y$	$-5/2\ d(4 + d^2)$
$y\ q$	$5/2\ d(4 + d^2)$
$q\ z$	$-6\ d(4 + d^2)$
$z\ q$	$6\ d(4 + d^2)$
$r\ r$	$-2 - 1/2\ d^2$
$s\ s$	$\frac{25}{4}\ d^2 + 25$
$y\ y$	$1/4\ (4 + d^2)(4\ d^2 + 9)$
$y\ z$	$3/2\ (3\ d^2 + 2)(4 + d^2)$
$z\ y$	$3/2\ (3\ d^2 + 2)(4 + d^2)$
$z\ z$	$3\ (4 + d^2)(d^2 + 2)(d^2 + 1)$

Table 4: The quadratic coefficients $A_{lm}(d)$ up to level 3. Omitted coefficients are zero.

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